



Oscillating flow in a stirling engine heat exchanger

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ABSTRACT

Three heat exchangers exist in modern Stirling engines: a heater, a cooler, and a regenerator. Here a study that deals principally with tubular heaters and coolers is carried out. The calculation procedure for the oscillating flow heat transfer is presented. Literature sources are studied to find the most suitable correlations by comparing them to each other and to the classical turbulent flow correlations encountered in the literature. The enhancement of heat transfer by means of a few circumferential slots inside the tubes and the pressure losses of oscillatory flow are discussed. Non-circular cross-section conduits with rectangular and triangular cross-sections are investigated and compared to the smooth circular tubes. The increment of the performance of an idealised Stirling engine with slotted heat exchanger tubes is compared to the case with smooth ones. The ratio of the gain in the shaft power and pumping losses is 2.22. The Carnot efficiency increment is 2.7%.

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1. Introduction

Convective heat transfer equations for oscillating flow encountered in Stirling engines have not been published until recent decades. The common practice has been that the turbulent correlations, e.g. the Dittus–Boelter equation [1,2], are used since ‘no correlations existed up to now to calculate the heat transfer coefficient and friction factor in oscillating flow’ [3]. Turbulence takes time to form and settle and the picture to date is quite complex when heating and pressure variation are added. In the fully turbulent and fully laminar range the steady unidirectional approximations are fairly good [4]. The laminar range is covered well in the latest papers in terms of oscillating convective heat transfer. In the literature survey several papers were found about heat transfer for the oscillating pipe flow encountered in Stirling engine heat exchangers. In this study they are compared to three classical unidirectional flow equations (Table 1).

As early as 1929 Richardson and Tyler [5] discovered the so-called ‘annular effect’ in an oscillatory flow in a pipe, i.e. the maximum velocity in an oscillatory flow occurs near the wall rather than at the centre of the pipe, as in the case with unidirectional steady flow. This applies especially at higher cycle frequencies [6]. Tang and Cheng (1993) [7] (Eq. (1), Table 1) employed multivariate

statistical analysis to obtain an experimental correlation equation for the cycle-averaged Nusselt number in terms of three similarity parameters: Re , Re_ω , and the dimensionless fluid displacement A_0 . Their experimental runs were performed for air in a heated pipe for the following ranges of parameters: $7 < Re < 7000$; $7 < Re_\omega < 180$, and $0.06 < A_0 < 2.21$. On the basis of an experimental and numerical study, Zhao and Cheng (1996) [8] derived Eq. (2), which consists of only two similarity parameters, Re_ω and A_0 . They plotted $Nu/A_0^{0.85}$ as a function of Re_ω ($0 < Re_\omega < 500$), while the dimensionless oscillating amplitude of fluid A_0 variable got the values 8.5, 15.3, 20.4, and 34.9. De Monte et al. (1996) [9] (3) made a comprehensive analytical study of oscillating heat transfer in Stirling machines regarding the inside and outside heat transfer of the heater, cooler, and regenerator and the effect of heat-exchange effectiveness on the performance of a Stirling engine. They applied the empirical correlation proposed by Tang and Cheng (1993) [7] and suggested that the correlation was valid under the following range of conditions: $11 < Re_{max} < 10,995$; $1.75 < Re_\omega < 45$. Walther et al. (1998) [6] performed a theoretical analysis to evaluate the influence of developing flow on the heat transfer associated with laminar oscillating pipe flow. Their heat transfer correlation (4) (the integral Nusselt number describing the averaged wall heat transfer as a function of the dimensionless oscillating frequency, the pipe length, and the bulk velocity) was plotted for the range: $1000 \leq Re_{max} \leq Re_{max}^{trans}$; $50 \leq Re_\omega \leq 900$; $0.2 \leq A \leq 900$, where $Re_{max}^{trans} = 400\sqrt{Re_\omega}$.

In this study Eqs. (1)–(4) were first tested by numerical calculations in a tubular heat exchanger and then compared to three

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Nomenclature			
A	cross-sectional area of moving element or flow	V_{SE}	swept volume of piston
A_o, A_R, A_ω	dimensionless amplitude of fluid displacement or stroke	v	mean velocity of flow
a_1, a_2	inside and outside width of rectangular duct	x	fluid or piston displacement, local coordinate
b_1, b_2	inside and outside height of rectangular duct	x_{max}	amplitude of fluid displacement
c	outer leg of triangular duct	<i>Greek symbols</i>	
C_f	Darcy friction factor	α	heat transfer coefficient
d	inner diameter of circular tube	Δ	difference
d_h	hydraulic diameter of duct or tube	δ_w	wall thickness
f	function in the correlation formula given by Petukhov and Popov	ε_1	entrance effect correction factor
G	total conductance	η_c	Carnot efficiency
L	tube length	λ	Fanning friction factor
Nu	Nusselt number	λ_w	conductance of wall
P	shaft power	Λ	relative amplitude of fluid displacement, critical length ratio
P_F	pumping losses	μ	dynamic viscosity
Pr	Prandtl number	ρ	density
Re	Reynolds number	τ_w	shear force at duct wall
q_m	mass flow rate	Φ	heat flow
q_v	volume flow rate	φ	phase angle, drag coefficient induced by sudden changes of flow by geometry
Re_m	Re average over half a cycle	ω	oscillation frequency
Re_ω	kinetic Reynolds number ($= \rho \omega d^2 / \mu$)	<i>subscripts</i>	
T	temperature	i	inner
T_H	heat source temperature	o	outer
T_L	coolant temperature	C	cooler
u_m	cross-sectional mean velocity	E	heater
u_{max}	amplitude of the cross-sectional mean velocity	DE	dead volume
$\bar{u}$	mean velocity of unidirectional flow	F	friction
V_E	momental expansion volume	SE	swept volume
V_{DE}	dead volume of expansion space	x	local

classical correlations (5)–(7) by Dittus and Boelter [10], Petukhov and Popov [11], and Gnielinski [12], respectively, meant for unidirectional turbulent developed flow but applied to the oscillating pipe flow conditions. Eq. (7) of Gnielinski [12] covers a large Re range: $3000 < Re < 10^6$.

The reciprocating flow in Stirling engines changes its direction constantly. During one cycle the laminar, transitional, and turbulent forms of flow modes occur. In this paper a high-frequency (8-Hz) low heat source temperature engine is considered. The review of these equations is performed in order to find tools to enhance the

heat transfer of oscillating flow by narrow boundary layers on the walls of heater and cooler tubes. In addition to the circular tubes, circumferentially slotted and non-circular rectangular and triangular tubes are studied to improve the performance of the Stirling engine that is being studied.

2. Assumptions and input values

In order to calculate the heat transfer in the heat exchangers the following assumptions are made according to de Monte et al.

Table 1
Oscillating and classical unidirectional heat transfer equations.

$Nu = -0.494 + 0.0777 \cdot \left(\frac{A_\omega}{1 + A_\omega} \right)^2 \cdot Re^{0.7} - 0.00162 \cdot Re^{0.4} Re_\omega^{0.8} \quad A_\omega = \frac{x_{max}}{L} = \frac{\pi \cdot u_m}{\omega \cdot L} = \frac{d \cdot Re}{L \cdot Re_\omega} \quad (1)$	
$Nu = 0.02 \cdot A_o^{0.85} \cdot Re_\omega^{0.58} \quad A_o = \frac{x_{max}}{d} \quad (2)$	
$Nu = -0.494 + 0.0777 \cdot \left(\frac{A_R}{1 + A_R} \right)^2 \cdot Re^{0.7} - 0.00162 \cdot Re^{0.4} \cdot (4 \cdot Re_\omega)^{0.8} \quad A_R = \frac{1}{4} \frac{d_h \cdot Re_{max}}{L \cdot Re_\omega} \quad (3)$	
$Nu = 5.78 + 0.00918 \cdot Re_\omega^{0.969} \cdot \Lambda^{-0.367} + 0.178 \left \frac{Re}{Re_{max}} \right ^{0.951} \cdot \Lambda^{-0.703} \cdot Re_\omega^{0.526} \quad \Lambda = \frac{1}{2} \frac{L \cdot Re_\omega}{d \cdot Re_{max}} \quad (4)$	
$Nu = 0.023 \cdot Re^{0.8} Pr^{0.4} \quad (5)$	
$Nu = \frac{(f/2) Re Pr}{(1 + 13.6 \cdot f) + (11.7 + 1.8 \cdot Pr^{-1/3}) \left(\frac{f}{2} \right)^{1/2} (Pr^{2/3} - 1)} \quad f = (3.64 \log Re - 3.28)^{-2} \quad (6)$	
$Nu = \frac{(f/8) \cdot (Re - 1000) \cdot Pr}{1 + 12.7 \cdot (f/8) \cdot (Pr^{2/3} - 1)} \quad 3000 < Re < 10^6 \quad f = (0.79 \cdot \ln Re - 1.64)^{-2} \quad 10^4 < Re < 5 \cdot 10^6 \quad (7)$	

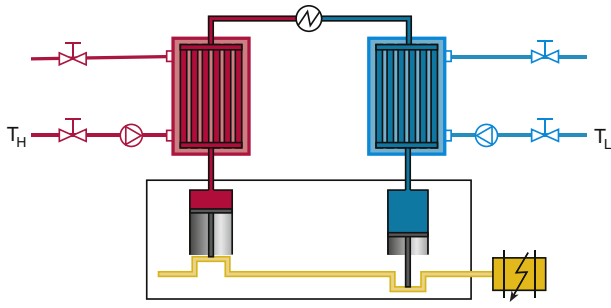


Fig. 1. Alpha-type Stirling engine.

(1996) [9]: 1) the working fluid temperatures in the heater and cooler are uniform and time-independent; 2) the ideal gas equation applies; 3) the cycle gas density in the heat exchangers is constant; 4) an ‘alpha’-type engine is considered; 5) the machine is working in cyclic steady-state operation, characterised by sinusoidal laws of motion of the piston. Input values are presented in the context of the calculations.

The alpha-type engine is one of three main types of Stirling engine arrangements. The alpha-type engine uses two pistons that mutually compress, expand, and move the working gas between hot and cold spaces (Fig. 1). See, for instance, Martini 1983 [13] for a more detailed description of the arrangements. The tubular heat exchangers in the engine that are studied are realised by counter-flow configuration.

3. Piston motion and dimensionless numbers

The time-averaged velocity in a Stirling engine is zero, or at least very low, since in a Stirling engine we do not have a pulsating flow, but an oscillating flow, meaning that the stationary component of the flow rate is zero.

The Schmidt theory is one of the isothermal calculation methods for Stirling engines [14]. For the alpha-type engine the momental expansion volume V_E in Eq. (8) is function of a swept volume of the piston V_{SE} and the dead volume of expansion space V_{DE} [14].

$$V_E = \frac{V_{SE}}{2} (1 - \cos\varphi) + V_{DE} \quad (8)$$

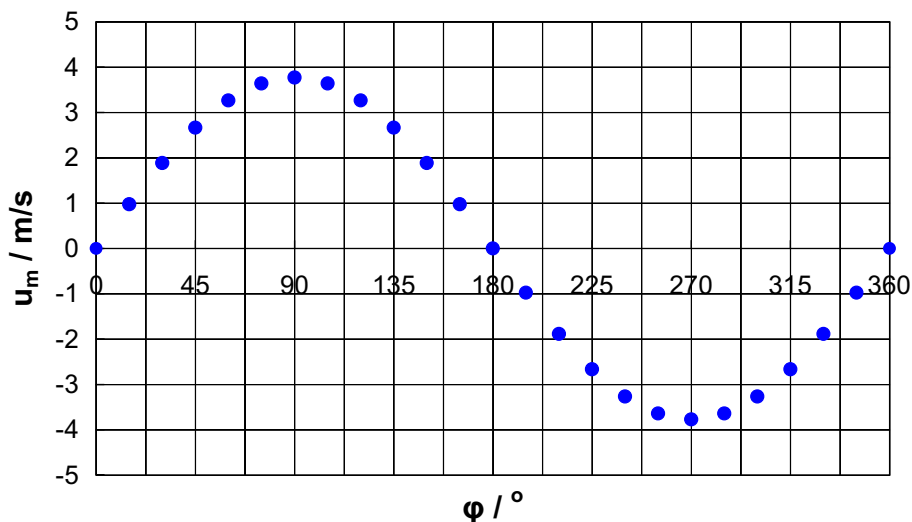


Fig. 2. Cross-sectional mean velocity u_m as a function of the piston crank angle.

where φ is the crank angle. By substituting $V = x \cdot A$ and $A_E = A_{SE} = A_{DE} = A$, we get

$$x_E \cdot A = \frac{x_{\max} \cdot A}{2} (1 - \cos(\omega t)) + x_{DE} \cdot A \quad (9)$$

Differentiating Eq. (9) with respect to time gives the cross-sectional mean velocity u_m

$$u_m = \frac{x_{\max} \cdot \omega}{2} \sin(\omega t) = \frac{x_{\max} \cdot \omega}{2} \sin\varphi \quad (10)$$

The phase angle $\varphi = \omega t$ of the cross-sectional mean velocity ranges from 0 to 2π . The amplitude of the cross-sectional mean velocity $u_{\max}$ (Eq. (11)) is obtained when $\varphi = \pi/2$. It is related to the amplitude of the fluid displacement $x_{\max}$ by the piston [8] (stroke, bore: 0.15 m, 0.3 m) and is the distance between the top and the bottom dead ends.

$$u_{\max} = \frac{x_{\max} \cdot \omega}{2} \quad (11)$$

Eq. (10) is used as the hydrodynamic boundary condition for the calculation of the heat transfer characteristics in the oscillating pipe flow. The mean velocity u_m is illustrated in Fig. 2 as a function of the crank angle (where $u_{\max}$ is 3.75 m/s).

By means of u_m and $u_{\max}$ we can calculate Re_m , $Re_{\max}$ and q_m and $q_{m \max}$. These parameters are encountered in the literature correlations (1)–(4) of the oscillating pipe flow.

Fig. 2 presents the cross-sectional mean velocity for piston. For the heater tubes that were studied (helium as working medium: 150 °C, 120 bar) the corresponding cross-sectional mean velocity amplitude $u_{\max}$ is 14 m/s and $Re_{\max} = 47,423$ at a frequency of 8 Hz because of the different face areas between the piston and the bundle of 670 tubes. According to de Monte et al. (1996) [9],

$$Re_m = \frac{2}{\pi} Re_{\max} \quad (12)$$

The Re ($=Re_m$) linked to the Eqs. (1) And (3)–(7) is based on the cross-sectional mean velocity u_m . It is a kind of average over half a cycle (Eq. (12)).

4. Calculation procedure

Eq. (2) by Zhao and Cheng (1990) [8] is used in demonstrating the calculation sequence. u_m is calculated from the Eq. (10), while ω

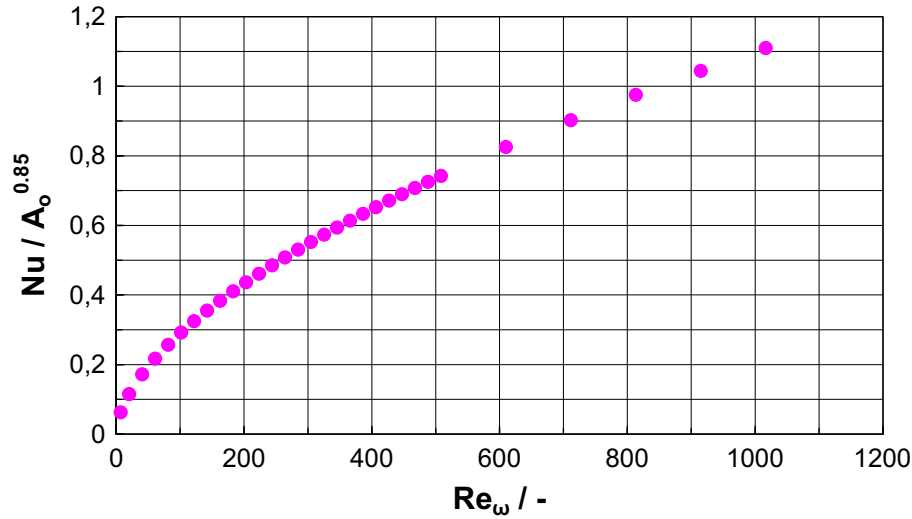


Fig. 3. $\overline{Nu}/A_0^{0.85}$ as a function of Re_ω .

and the stroke $x_{\max}$ are constants of 50.3 1/s (~ 8 Hz) and 0.15 m respectively. The phase (crank) angle of the piston φ and u_m vary according to Fig. 2. Re_m also varies but Re_ω has a constant value of 1022. The thermophysical properties of helium at a pressure of 120 bar [15] are used to calculate the density ρ , the dynamic viscosity μ , the Prandtl number Pr , and the conductivity λ that appear in the dimensionless numbers. By using the mean velocity amplitude of 14 m/s encountered in the heated tubes ($u_{\max}$), we can calculate the amplitude of the fluid displacement in the tubes ($x_{\max}$) from Eq. (11). The dimensionless oscillating amplitude of fluid A_0 in the pipes is calculated through the pipe diameter d and $x_{\max}$. Finally Eq. (2) gives the mean value of 52.6 W/(m²K) for Nu .

According to Fig. 2, we see that u_m receives its maximum value when the phase angle is 90°. By changing the frequency $0 < f \leq 8$ Hz ($0 < \omega \leq 50$ 1/s) and keeping φ as a constant ($\pi/2$), we can plot $Nu/A_0^{0.85}$ as a function of Re_ω [8], as presented in Fig. 3.

The plotted $Nu/A_0^{0.85}$ as a function of Re_ω is well compatible with Zhao and Cheng 1996 [8] but it is extended to the range $0 < Re_\omega \leq 1022$, while it was originally presented for the range $\sim 0 \dots 500$.

In this paper we will plot the graphs for Nu as a function of Re_ω .

5. Heat transfer

Pressurised hot water (T_H) on the shell side is used as a heat source, as shown in Fig. 1. For the heat transfer calculations a constant temperature of 150 °C and a pressure of 120 bar inside the heater tubes were selected for the working medium, helium. The Nu numbers of correlations (1)–(7) on the helium side are plotted in Fig. 4 as a function of Re_ω .

Fig. 4 indicates that when the frequency of the oscillation is small some of both correlation groups (oscillating and unidirectional turbulent flow) have almost the same slopes and the graphs are almost convergent. As the Re_ω increases the slope of the laminar oscillating correlations starts to decrease gradually.

Eq. (2) from Zhao and Cheng (1996) [8] gives the second largest Nu values of the oscillating graphs. It is a conservative estimate of the heat transfer rate for a reciprocating flow in a pipe of infinite length. Here only two dimensionless parameters, A_0 and Re_ω , are included. The other oscillation flow Eqs. (1), (3), and (4), which include the classical Re number as a third non-dimensional term (commonly used in the unidirectional pipe flow) give smaller Nu values.

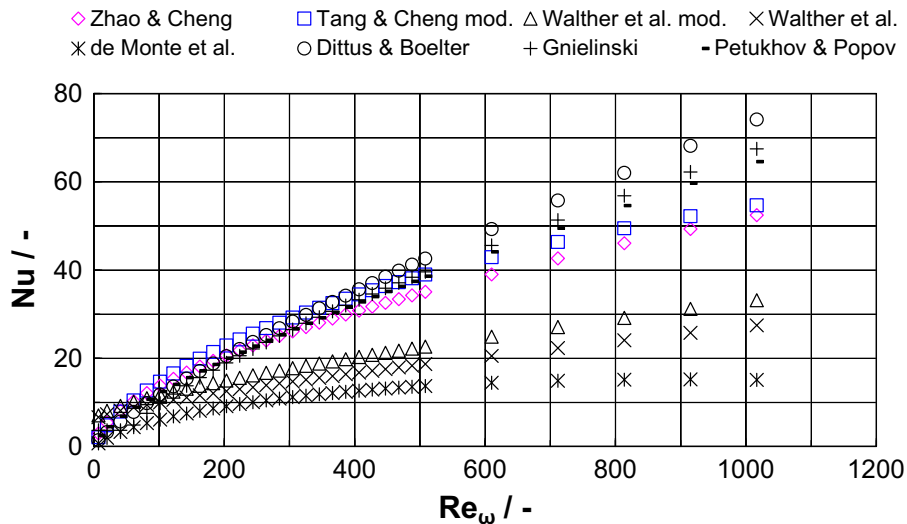


Fig. 4. Nu numbers of correlations studied as a function of Re_ω .

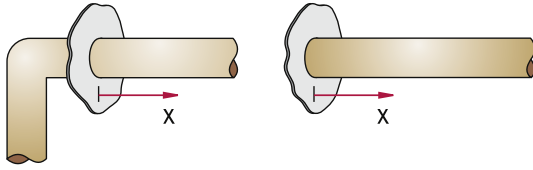


Fig. 5. The 90° elbow (left) and the open-end 90° edge (right).

By comparing the non-dimensional oscillating amplitudes A_R and A_ω in the sources de Monte et al. (1996) [9] and Tang and Cheng (1993) [7], it is evident that $Re = Re_{max}$. The modified (mod) diagrams are arrived at by this assumption. Under this circumstance the highest oscillating Nu values were plotted from Eq. (1). Otherwise, we did not get the positive Nu values from that equation.

We did not find the oscillating heat transfer correlations for the turbulent flow. However, we can conclude that unidirectional turbulent correlations give slightly higher dimensionless heat transfer coefficients at the higher frequencies (Re_ω numbers) than the calculated coefficients from the oscillating equations of Zhao and Cheng (2) and the modified Tang and Cheng (1).

Ahn and Ibrahim (1992) [16] conducted two-dimensional analysis of oscillating gas flow in laminar, transition and turbulent regimes inside Stirling engine heat exchangers. Numerical data were compared to the experimental results of other investigators with good accuracy in laminar and turbulent flow regimes. They reviewed several criteria for the critical Reynolds number for oscillating pipe flow. The onset of the turbulence and frictional losses in an oscillatory turbulent pipe flow is discussed in another source, Zhao and Cheng (1996) [17]. The oscillating flow is in the turbulent region when $A_0 \sqrt{Re_\omega} \geq 961$ [17]. At the 8-Hz frequency that was studied we get 2966 for such a product (of $A_0 = 93$ and $Re_\omega = 1017$) and the flow is turbulent.

6. Heat transfer enhancement

In this work several traditional and some up-to-date heat transfer enhancement methods, e.g. regarding the laminar convective heat transfer problem as a conduction problem [18], were gone through. However, here only the effect of circumferential slots in a circular pipe and the enhancement of heat transfer by non-circular square and triangular ducts are reported.

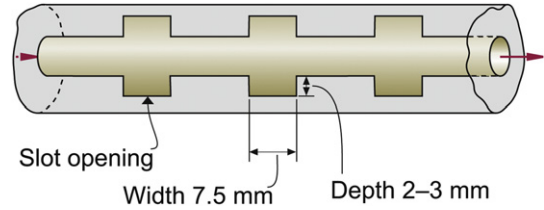


Fig. 7. The heater tube with three circumferential slots.

According to Mills (1995) [12], turbulent flows with simply defined hydrodynamics at a pipe entrance are seldom encountered in engineering practice. Most often there is a sharp 90° edge, a bend, or an elbow (Fig. 5).

The corresponding hydrodynamic entrance lengths vary from about 10 to 15 diameters, where no large-scale eddies are present, to about 30–40 diameters, where there are large-scale eddies [12]. The effect of entrance configuration on the average heat transfer for the turbulent tube flow is presented in the Fig. 6.

It is visible in the Fig. 6 that the open-ended 90° edge and 90° elbow give the largest eddies and the most long-range average Nu enhancement. The observation that ‘the edges improve the inside heat transfer’ in the tubular heat exchanger was studied more carefully.

It is suggested that a few sharp edges inside the pipe could produce a greater local Nu number and a narrower laminar boundary layer along the entire length compared to the smooth pipe. The axial slots (grooves) are used e.g. in the annular ducts of electric machines on the stator and/or rotor surfaces in order to enhance the heat transfer [19].

The heater tube length (170 mm) was divided into four similar parts by means of three circumferential slots (Fig. 7).

The depth and width of one slot opening are assumed to be 2–3 mm and 7.5 mm, respectively. In this case the effective heat transfer length is still 170 mm, even if the total pipe length with slots is increased to 192.25 mm.

A factor and numerical fit was made for the open-ended 90° edge entrance effect. Eq. (2) from Zhao and Cheng (1996) [8] was multiplied by this factor. The effects in the entrance and after the sequential slots in the straight plain tube were assumed to be identical. In addition this was compared to the factor ε_1 in Eq. (13) of Sukomel et al. (1987) [20] (Fig. 8).

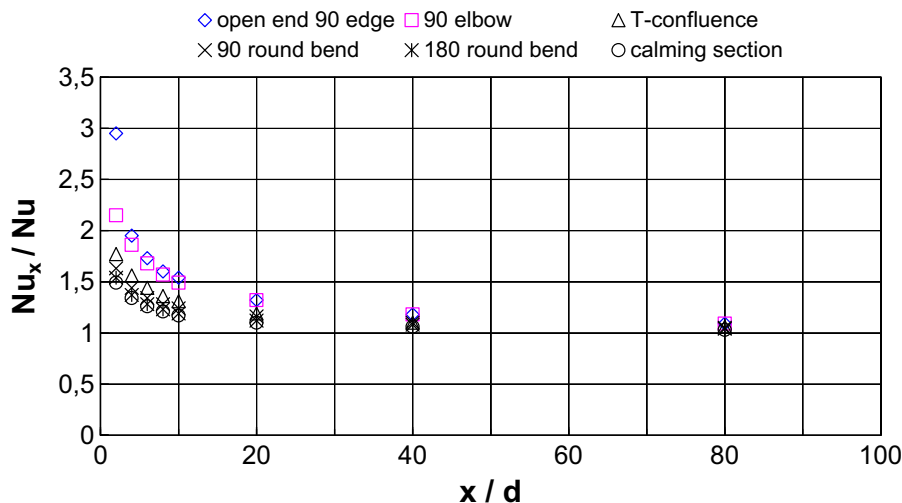


Fig. 6. The effect of entrance configuration on the average heat transfer in the turbulent tube flow.

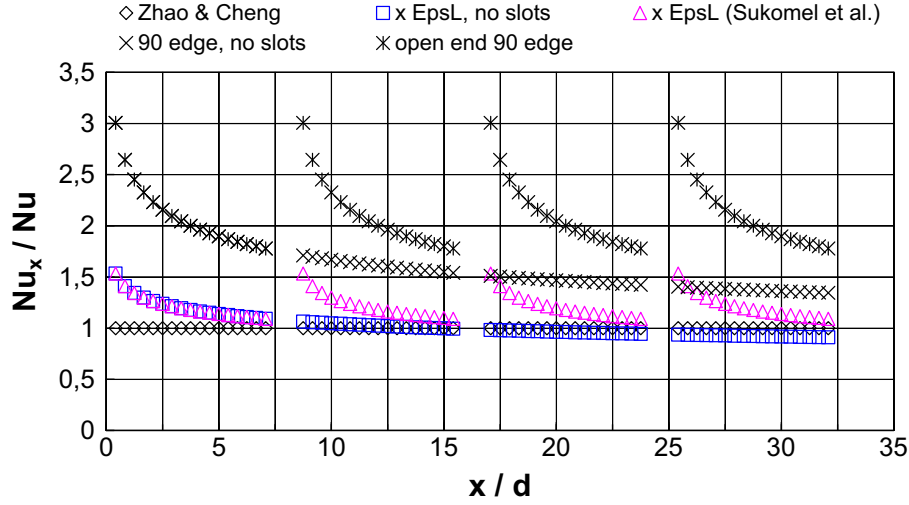


Fig. 8. Local Nu_x enhancement divided by cycle-averaged Nusselt number Nu .

$$Nu_x = 0.022Re^{0.8}Pr^{0.43}\epsilon_1 \quad (13)$$

$$\epsilon_1 = 1.38(x/d)^{-0.12}$$

In Fig. 8 we can see the mean heat transfer from Zhao and Cheng (2) and the entrance effect without and with circumferential slots. The effect of the factor ϵ_1 [20] with slots compared to the mean heat transfer looks promising. It is selected to account for the change in the heat transfer coefficient in the initial section of the tube and after the slots.

The performance of the selected non-circular cross-sections of rectangular and triangular tubes was investigated and compared to that of the circular tube. Eqs. (14)–(16) for the total conductance G were derived for the circular, rectangular, and equilateral triangular tubes.

$$\frac{l}{G} = \frac{1}{\alpha_i \pi d_i} + \frac{1}{\frac{\lambda_w}{\delta_w} \pi \frac{d_i + d_o}{2}} + \frac{1}{\alpha_o \pi d_o} \quad (14)$$

l is the pipe length, λ_w and δ_w the conductance and thickness of the wall. For the circular pipe d_i and d_o are the inner and the outer pipe diameters and α_i and α_o the corresponding heat transfer coefficients at the surfaces.

$$\frac{l}{G} = \frac{1}{2\alpha_i(a_1 + a_2)} + \frac{1}{2\lambda_w \frac{a_1 + a_2 + b_1 + b_2}{b_1 - a_1}} + \frac{1}{2\alpha_o(b_1 + b_2)} \quad (15)$$

For the rectangular duct a_1 and a_2 (and b_1 and b_2) are the inside (and outside) width and height of the duct.

$$\frac{l}{G} = \frac{1}{3\alpha_i \left(c - \frac{2\delta_w}{\tan 30^\circ} \right)} + \frac{1}{3\lambda_w \left(c - \frac{\delta_w}{\tan 30^\circ} \right)} + \frac{1}{3\alpha_o c} \quad (16)$$

For the triangular equilateral duct c is the outer leg of the triangle and δ_w the wall thickness.

It was found out that the smooth non-circular square and triangular tubes have greater free flow and exterior areas than the circular tube with an equivalent hydraulic diameter [4]. Hence, for the equivalent heat flow magnitude the required tube lengths of square and triangular ducts were 21% and 39% shorter than the length of the circular tube. However, the dead volume caused by the heat exchanger remains constant.

7. Pressure losses

The internal flow pressure loss of the smooth and slotted tubes was studied and the loss of the smooth tubes of a non-circular cross-section (rectangular and triangular) was compared to the loss in the circular tube.

Two different friction factor definitions are in common use in the literature [21], the Darcy friction factor C_f and the Fanning friction factor λ :

$$C_f = \frac{2\tau_w}{\rho \bar{u}} = \frac{1}{4}\lambda \quad (17)$$

The losses are calculated as follows:

$$\Delta p = \left(\lambda \frac{L}{d_h} + \Sigma \varphi \right) \frac{\rho v^2}{2} \quad (18)$$

The drag coefficient φ includes all sudden changes in the flow as a result of the geometry. d_h is the hydraulic diameter.

Eq. (18) applies all kinds of flow patterns, e.g. laminar and turbulent. The drag coefficient λ depends on the Reynolds number for the flow within the duct. The hydraulic diameter d_h allows Eq. (18) to be applied to any given cross-section [22]. The plot of λ is presented in the source [22] for tubes of various cross-sections as a function of the Reynolds number.

The friction coefficient C_f of unidirectional flow can be calculated from the correlation (19) for laminar and from the correlation (20) for the turbulent flow [23].

$$C_f = \frac{64}{Re}, \quad Re < 2000 \quad (19)$$

$$C_f = 0.079Re^{-1/4}, \quad Re < 10^5 \quad (20)$$

The friction coefficient of a cyclically turbulent oscillatory flow was measured by Zhao and Cheng in 1996 (Eq. (21)) [17].

$$C_f = \frac{1}{A_o} \left(\frac{76.6}{Re_\omega^{1.2}} + 0.40624 \right) \quad (21)$$

which is valid in the range of $81 \leq Re_\omega \leq 540$ and $54.4 \leq A_o \leq 113.5$.

The laminar and turbulent drag coefficients for unidirectional tube flow were plotted when $\lambda = 4C_f$ as a function of Re_ω . This curve

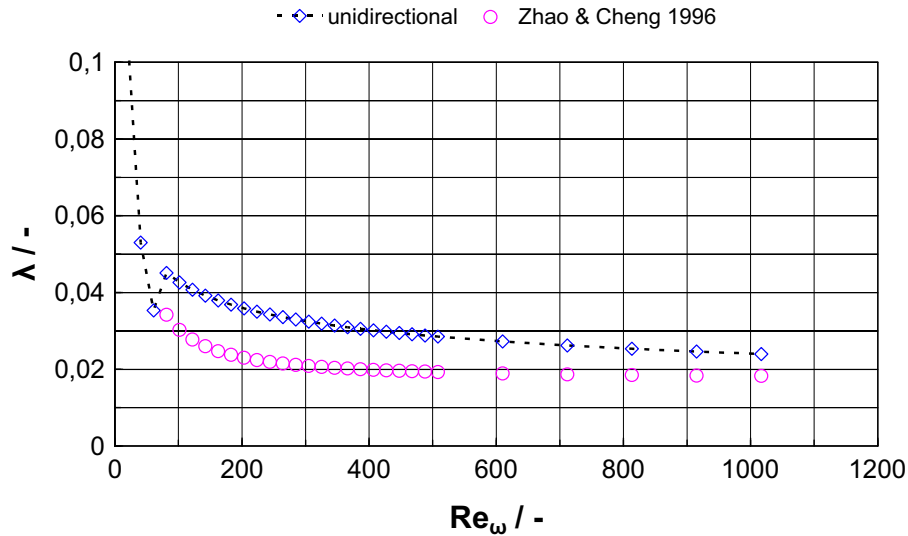


Fig. 9. The laminar and turbulent drag coefficients for unidirectional tube flow compared to the drag coefficient of a cyclically turbulent oscillatory flow.

is compared to the friction coefficient graph of the cyclically turbulent oscillatory flow in Fig. 9.

The oscillatory Eq. (21) for the turbulent flow gives slightly lower values for the friction coefficient than the unidirectional friction Eq. (20) (Fig. 9). In the studied heat exchanger that was studied $A_0 = 93.3$ and $Re_w = 1016.8$ (~ 8 Hz) and the flow is turbulent, after Zhao and Cheng [17].

The oscillating pressure loss for the circumferentially slotted circular tube (3 slots, 2 mm depth) is 8.17 times higher than for the smooth tube at a frequency of 8 Hz. The friction loss towards one tube is small (~ 1 W), however.

The pressure losses of unidirectional and oscillating flow for the smooth circular and non-circular tubes are presented in the Fig. 10. It is visible that unidirectional flow in the tube retains larger losses than the oscillating case. The length of the circular tube is 0.17 m and the inner diameter is 0.006 m.

The square and triangular tubes correspondingly have only marginally lower losses (as a result of the diminished pipe lengths that give equivalent heat transfer) compared to the smooth circular tube.

Friction coefficients and Nusselt numbers for fully developed laminar flow are presented for a large variety of tube shapes in the source Rosenhow et al. (1985) [24]. In all cases the hydrodynamic diameter is used as the characteristic length in the Reynolds and Nusselt numbers. The friction $C_f Re$ and Nu values of the regular polygonal tubes decrease in a gently sloping manner as the number of sides decreases.

Bejan and Lorente (2006) suggest that the round tube shape is the best regarding the laminar flow resistance. However, nearly round shapes such as hexagonal and square tubes perform almost as well after the constructal law. Even if the duct cross-section is imperfect – that is, with features such as sharp corners, which concentrate fluid friction – its performance is already as good as it can be [25].

8. Performance with slotted tubes

The thermodynamic performance of an idealised Stirling engine was studied by comparing the heat transfer between circumferentially slotted and smooth circular tubes in the heat

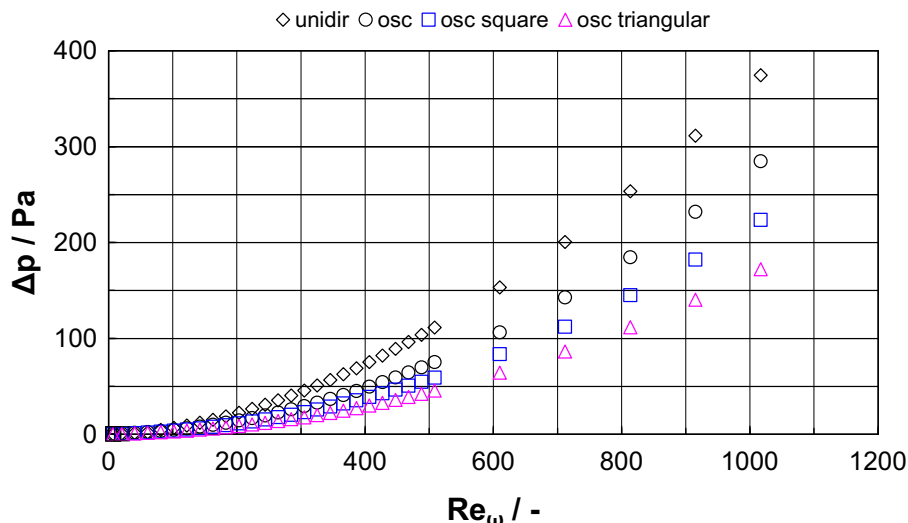


Fig. 10. Pressure losses in one smooth tube: unidirectional and oscillating flow in the circular tube and the oscillating flow in non-circular square and triangular tubes.

exchangers. The identical heater and cooler by the geometry for the engine that was studied were roughly dimensioned. The effect of the slots was considered from two points of view.

First, the difference in the shaft power was calculated according to Eq. (22) when comparing slotted and smooth tubes in the heater.

$$\Delta P = \eta \cdot \Delta \Phi_E \quad (22)$$

where ΔP is the difference in the shaft power when using slotted and smooth tubes, $\Delta \Phi_E$ the corresponding heat flow difference in the heater and η the efficiency ($= 0.22$) of the selected Stirling engine. In this first study the logarithmic temperature difference ΔT_E between helium and water in the heater was supposed to remain constant. The heater heat flow Φ_E was increased as a result of the rise in the conductance G_E and the inside heat transfer coefficient α_i (Eqs. (14) and (23)) while using the circumferential slots compared to the smooth case.

$$\Phi_E = G_E \Delta T_E \quad (23)$$

The Φ_E values were calculated for the slotted and smooth heat exchanger tubes used in the Eq. (22).

The corresponding (oscillating flow) pressure losses were calculated by Eqs. (17), (18), (21) and Eq. (24)

$$\Delta P_F = q_V \cdot \Delta p \quad (24)$$

for a heater consisting of 670 tubes. The drag coefficients φ was taken from the source [26]. As a result we got the power ratio (Eq. (25))

$$\frac{\Delta P}{\Delta P_F} = \frac{1336,8 \text{ W}}{603 \text{ W}} = 2.22 \quad (25)$$

Here the factor ε_1 (Eq. (13)) [20] together with Eq. (2) from Zhao and Cheng [8] was used to calculate the (arithmetic) mean heat transfer coefficients for the slotted pipes (3 slots, depth 2 mm) and the difference $\Delta \Phi_E$ and ΔP .

From Eq. (25) we can see, on the basis of previous assumptions, that the gain in the shaft power of the Stirling engine compared to the increase in the pumping losses is 2.22 when these circumferentially slotted tubes are used. The same ratio is 1.94 for the tubes consisting of 3 slots with a depth of 3 mm.

We also considered the efficiency increment. In this case we kept the cooler and heater heat flows Φ_C and Φ_E and the shell-side (water) temperatures constant ($\Phi_{\text{slotted pipe}} = \Phi_{\text{smooth pipe}}$). The subscript C refers to the cooler. As a result of the slots in the tube both G_C and G_E increase and the logarithmic ΔT_C and ΔT_E are reduced compared to the smooth tube (Eq. (23)). Hence the new working fluid temperatures T_E and T_C can be solved for the helium in the slotted heater and cooler and the Carnot efficiency (Eq. (26)) [27].

$$\eta_C = 1 - \frac{T_C}{T_E} \quad (26)$$

The increment in the efficiency of the Stirling engine can be calculated as

$$\Delta \eta_C = \frac{\eta_{C \text{ slotted}} - \eta_{C \text{ smooth}}}{\eta_{C \text{ smooth}}} \quad (27)$$

where $\eta_{C \text{ smooth}}$ is the solved Carnot efficiency with a smooth heater and cooler tubes and $\eta_{C \text{ slotted}}$ is the calculated efficiency with slotted tubes.

By using the heat transfer enhancement by the factor ε_1 [20] together with Eq. (2) from Zhao and Cheng [8] to calculate the working fluid heat transfer coefficients in the slotted cooler and the

heater (and conductance G_C and G_E), we got a T_E increase of 2.5 °C, a T_C fall of 2.3 °C, and $\Delta \eta \sim 0.0272$. It means a 2.72% increment in the efficiency of the Stirling engine by using these circumferentially slotted tubes instead of smooth ones.

9. Conclusions

Earlier fully laminar and turbulent steady unidirectional equations were used as an approximation for convective heat transfer in a Stirling engine. Hitherto the literature has dealt with the laminar oscillating correlations. In this study the existing laminar equations are compared and extended to the turbulent range. It is noticed that some of them give almost the same Nu values as turbulent unidirectional equations.

The salient parameters are the amplitude of the cross-sectional mean velocity, the kinetic Reynolds number, and the dimensionless oscillating amplitude. The equations based on these two latest parameters give Nu values that are almost as high as the traditional equations in the turbulent range.

Heat transfer enhancement is studied with a few circumferential slots inside the smooth tubes. The effect of edges in the entrance and the sequential slots is assumed to be identical. In addition, non-circular rectangular and triangular tubes are investigated. The square and triangular tubes have greater free flow and exterior areas than the circular tube with the same hydraulic diameter. Hence the duct lengths are also shorter. However, the dead volume in the heat exchanger remains constant.

The friction coefficient of oscillating flow is lower than in the unidirectional approximation. The frictional losses in slotted tubes are greater but with square and triangular tubes they are marginally lower than with smooth circular ones.

The increment of the performance in the idealised Stirling engine with slotted heat exchanger tubes in oscillating flow is compared to smooth tubes. The ratio of the gain in the shaft power and pumping losses is 2.22 compared to smooth tubes. The Carnot efficiency increment is 2.72%.

With these simplified examinations it can be concluded that methods (few edges or circumferential slots) that form narrower boundary layers along the length of the tube but do not significantly increase the dead volume or pressure loss are recommended in the design of the heater and cooler of the Stirling engine.

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